Adaptivity Gap for Influence Maximization with Linear Threshold Model on Trees

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Abstract. We address the problem of influence maximization within the framework of the linear threshold model, focusing on its comparison to the independent cascade model. Previous research has predominantly concentrated on the independent cascade model, providing various bounds on the adaptivity gap in influence maximization. For the case of a (directed) tree (in-arborescence and out-arborescence), [CP19] and [DPV23] have established constant upper and lower bounds for the independent cascade model.

However, the adaptivity gap of this problem on the linear threshold model is not so extensively studied as on the independent cascade model. In this study, we present constant upper bounds for the adaptivity gap of the linear threshold model on trees. Our approach builds upon the original findings within the independent cascade model and employs a reduction technique to deduce an upper bound of $\frac{4e^2}{e^2-1}$ for the in-arborescence scenario. For out-arborescence, the equivalence between the two models reveals that the adaptivity gap under the linear threshold model falls within the range of $[\frac{e}{e-1}, 2]$, as demonstrated in [CP19] under the independent cascade model.

1 Introduction

The *influence maximization problem*, initially introduced in [DR01,RD02], is a well-known problem that lies at the intersection of computer science and economics. It focuses on selecting a specific number of agents, referred to as seeds, in a social network to maximize the number of agents influenced by them. To analyze this problem mathematically and formally, social networks are represented as weighted graphs, where vertices correspond to agents and edges represent their connections, with each edge assigned a weight indicating the strength of the connection. The *independent cascade model* [KKT03] and the *linear threshold*

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model [KKT03] are two prominent diffusion models that have received significant attention in previous studies. These models have been applied to various fields such as viral marketing, meme usage, and rumor control.

More recently, the *adaptive influence maximization* problem has gained considerable attention. Unlike the original setting where all seeds are selected at once, the adaptive version allows seeds to be selected based on observations of the propagation of previously chosen seeds. Of particular interest are two feedback models [GK11], namely *myopic feedback* and *full adoption feedback*. When considering myopic feedback, only the status of the seeds' neighbors can be observed. Conversely, the full adoption feedback allows the whole propagation process of previously selected seeds to be considered when selecting the next seed. While the introduction of adaptive seed selection might enhance the influence of the seed set, it also presents significant technical challenges. Therefore, it becomes imperative to evaluate the benefits of adaptivity, which is measured by the *adaptivity gap*. The adaptivity gap is informally defined as the supremum value of the ratio between the optimal influence spread of an adaptive policy and a non-adaptive one. It provides insights into the performance improvement achieved by the adaptive strategy and gives us a taste of whether it is worth the effort to develop the adaptive strategy for the problem.

Regarding the adaptivity gap, a number of previous works have explored this concept in the context of the independent cascade model [CP19,DPV23, PC19]. In [CP19], the adaptivity gap for the independent cascade model with full adoption feedback was studied for certain families of influence graphs. It was demonstrated that the adaptivity gap lies in the range of $[\frac{e}{e-1}, \frac{2e}{e-1}]$ for in-arborescence, $[\frac{e}{e-1}, 2]$ for out-arborescence, and exactly $\frac{e}{e-1}$ for bipartite graphs. Another recent work [DPV23] improved upon these results by providing a tighter upper bound of $\frac{2e^2}{(e^2-1)}$ for the adaptivity gap of in-arborescence. Furthermore, this work established an upper bound of $(\sqrt[3]{n}+1)$ for general graphs, where n stands for the number of vertices in the graph. For the myopic feedback setting, it has been proved in [PC19] that the adaptivity gap for the independent cascade model with myopic feedback is at most 4 and at least $\frac{e}{e-1}$. However, despite the progress made in analyzing the adaptivity gap for the independent cascade model, to the best of our knowledge, no existing results are available on the adaptivity gap for the linear threshold model.

1.1 Our Results

In this work, we give an upper bound for the adaptivity gap for in-arborescence under the linear threshold model as follows.

Theorem 1. *The adaptivity gap AG_{LT} for in-arborescence under the linear threshold model is no more than $\frac{4e^2}{e^2-1}$.*

Also, for out-arborescence, the linear threshold model is equivalent to the independent cascade model since each vertex has at most in-degree 1. Thus, the results under the independent cascade model for out-arborescence given by [CP19] can be also used in the linear threshold model (Table 1).

Theorem 2. *The adaptivity gap AG_{LT} for out-arborescence under the linear threshold model satisfies that $AG_{LT} \in [\frac{e}{e-1}, 2]$.*

Table 1. The previous results and the results of this paper are summarized in the table. New results of this paper are in blue.

Diffusion Model	Feedback Model	Graph Family	Lower Bound of Adaptivity Gap	Upper Bound of Adaptivity Gap
Independent Cascade	Full adoption feedback	In-arborescence	$\frac{e}{e-1}$	$\frac{2e^2}{(e^2-1)}$
		Out-arborescence	$\frac{e}{e-1}$	2
		Bipartite graphs	$\frac{e}{e-1}$	$\frac{e}{e-1}$
		General graphs		$(\sqrt[3]{n}+1)$
	Myopic feedback	General graphs	$\frac{e}{e-1}$	4
Linear Threshold	Full adoption feedback	In-arborescence		$\frac{4e^2}{e^2-1}$
		Out-abborescence	$\frac{e}{e-1}$	2

1.2 Related Works

The influence maximization problem was initially proposed in [DR01] and [RD02]. Subsequently, the two most extensively studied diffusion models, namely the independent cascade model and the linear threshold model, were introduced in [KKT03], which also demonstrated their submodularity. For any submodular diffusion model, the greedy algorithm is shown to obtain a $(1-1/e)$-approximation to the optimal influence spread [NWF78, KKT03, KKT05, MR10]. A later work [STY20] shows that the approximation guarantee of the greedy algorithm for the influence maximization problem under the linear threshold model is asymptotically $(1-1/e)$.

The adaptive influence maximization problem is first considered in [GK11]. The results relevant to adaptivity gaps under the independent cascade model have been discussed before, and we discuss further related work here. Later, Asadpour and Nazerzadeh studied the adaptivity gap for the problem of maximizing stochastic monotone submodular functions [AN16].

The adaptivity gap compares the optimal adaptive solution to the optimal nonadaptive solution. Motivated by that the inapproximability of the influence maximization problem [KKT03, ST20] and the fact that most influence maximization algorithms are based on greedy, the concept of greedy adaptivity gap is introduced in [CPST22], which depicts how much adaptive greedy policy would outperform its non-adaptive counterpart. This work also showed that the greedy adaptivity gap is at least $(1 - 1/e)$ for an arbitrary combination of diffusion and feedback models.

2 Preliminaries

2.1 Linear Threshold Model

In the *linear threshold model* (LT), we have a weighted directed graph called the influence graph $G = (V = [n], E, \{p_{u,v} \mid (u, v) \in E\})$, satisfying $\sum_u p_{u,v} \leq 1$. Fix a seed set $S \subseteq V$, the diffusion process in the LT model is defined as follows. Define the current activated vertex set T, and initialize $T = S$. Before the diffusion process starts, every vertex first independently samples a value $a_i \in [0, 1]$ uniformly at random. In each iteration, if a non-activated vertex x satisfies that $\sum_{u \in T} p_{u,x} \geq a_x$, it will be activated and let $T = T \cup \{x\}$. The diffusion process terminates when there is no more activated vertex in an iteration.

It is mentioned in [KKT03] that the LT model has another equivalent interpretation (Fig. 1). Fix a seed set $S \subseteq V$.

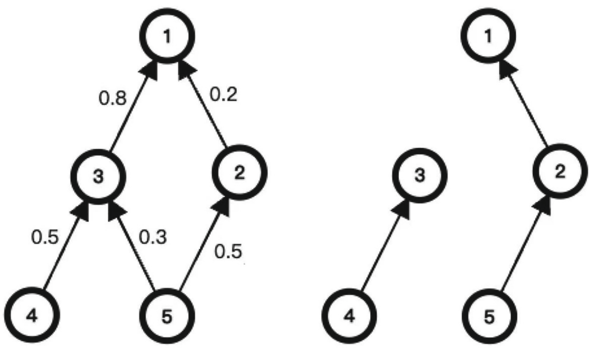

Fig. 1. The above pictures are an example of weighted directed influence graph and an instance of its live-edge graph. In the LT model, the right live-edge graph appears for a probability $0.5 \cdot 0.5 \cdot 0.2 = 0.05$. However, in the IC model, the appearing probability is $0.5 \cdot 0.7 \cdot 0.5 \cdot 0.2 \cdot 0.2 = 0.007$. Readers can find the differences between the two models in this example.

Then sample a *live-edge graph* $L = (V, L(E))$ of G, which is a random graph generated from the base graph G as follows. For each vertex i, sample at most one in-edge, where the edge (u, i) is selected with probability $p_{u,i}$, and add this edge (if exists) to $L(E)$. In this case, the diffusion process will activate all the vertices that can be reached from S. Given a live-edge graph L, use $R(S, L)$ to denote all the vertices activated at the end of this process. Given a seed set S, the expected influence spread of S is defined as $\sigma(S) := \mathbf{E}_L[|R(S, L)|]$.

2.2 Independent Cascade Model

The *independent cascade model* (IC) also involve a weighted directed influence graph $G = (V = [n], E, \{p_{u,v} \mid (u, v) \in E\})$. Before the beginning of the propagation, a *live-edge graph* $L = (V, L(E))$ is sampled. The sampling of the live-edge graph in the IC model is simpler than that of the LT model. Each edge $e \in E$ appears in $L(E)$ independently with probability p_e. When an edge is present in the live-edge graph, we say that it is *live*. Otherwise, we say that it is *blocked*. Denote the set of all possible live-edge graphs by $\mathcal{S}$, and the distribution over $\mathcal{L}$ by $\mathcal{P}$. Given a seed set $S \subseteq V$, the vertices affected, denoted by $\Gamma(S, L)$ is exactly the set of vertices reachable from S in the live-edge graph L. The influence reach of a certain seed set on a live-edge graph $f : \{0, 1\}^V \times \mathcal{L} \to \mathbf{R}^+$ is defined as the number of affected vertices, i.e., $f(S, L) = |\Gamma(S, L)|$. Then we define the influence spread of a seed set $\sigma(S)$ to be the expected number of affected vertices at the end of the diffusion process, i.e., $\sigma(S) = \mathbf{E}_{\mathcal{L}\sim\mathcal{P}}[f(S, L)]$.

2.3 Non-adaptive Influence Maximization

The *non-adaptive* influence maximization problem is defined as a computational problem that, given an influence graph G and an integer $k \geq 1$, we are asked to find a vertex set S satisfying that $|S| = k$ and maximizing $\sigma(S)$. Use $\mathrm{OPT}_N(G, k)$ to denote the maximal $\sigma(S)$ under graph G and parameter k. The subscript "N" stands for "non-adaptive", which is in contrast with the "adaptive" model defined in the next section.

2.4 Adaptive Influence Maximization

Compared with non-adaptive influence maximization problem, the *adaptive* setting allows to activate the seeds sequentially and adaptively in k iterations. One can first choose a vertex, activates it, and see how it goes. After observing the entire diffusion process of the first vertex, we can change their strategy optimally adaptive to the diffusion process. Similarly, the choices of the following vertices are based on the previous observation. We consider the *full-adoption* feedback model, which means the adaptive policy observes the entire influence spread from the previous chosen vertices.[1]

An *adaptive policy* can be formally defined as follows. Given a live-edge graph L, the realization $\phi_L : V \to 2^V$ denotes a function from a vertex to a vertex set. For a fix vertex v, define $\phi_L(v) := R(v, L)$, i.e., the set of vertices activated by v under the live-edge graph L. Given a subset S of V satisfying that $|S| = k$,

[1] Another commonly considered model is called the *myopic* feedback model, where only one iteration of the spread can be observed.

define the partial realization $\psi : S \to 2^V$ restricted to S to be the part of some realization, which can be used to represent the graph observed by the player at some point of the adaptive algorithm. For a fixed partial realization, let its domain (the chosen seed vertices) be $\text{dom}(\psi) := S$, let $R(\psi) = \cup_{v \in S} \psi(v)$, and let $f(\psi) = |R(\psi)|$. A partial realization ψ' is called a sub-realization of another partial realization ψ if and only if that $\text{dom}(\psi') \subseteq \text{dom}(\psi)$ and $\psi'(v) = \psi(v)$ for any $v \in \text{dom}(\psi')$.

2.5 Adaptivity Gap

The adaptivity gap for the LT model is defined as follows

$$AG_{\text{LT}} = \sup_{G,k} \frac{\text{OPT}_A^{\text{LT}}(G,k)}{\text{OPT}_N^{\text{LT}}(G,k)},$$

where $\text{OPT}_A^{\text{LT}}(G,k)$ is the optimal influence spread with a k-vertex seed set on graph G in the adaptive setting, and $\text{OPT}_N^{\text{LT}}(G,k)$ is its counterpart in the non-adaptive setting.

Similarly for the IC model, the adaptivity gap can be defined as follows

$$AG_{\text{IC}} = \sup_{G,k} \frac{\text{OPT}_A^{\text{IC}}(G,k)}{\text{OPT}_N^{\text{IC}}(G,k)}.$$

3 Adaptivity Gap for In-Arborescence

An *in-arborescence* is a directed graph $G = (V, E)$ that can be constructed by the following process: fix a rooted tree $T = (V, E')$, and add edge (u, v) if v is the parent of u in T. An upper bound for AG_{IC} for in-arborescence is given by [DPV23]. This bound also plays an essential role in our proof of the constant upper bound for AG_{LT} for in-arborescence.

We prove the following theorem:

Theorem 3. *$AG_{LT} \leq \frac{4e^2}{e^2-1}$ for in-arborescence.*

The key technique is to reduce the influence maximization problem in the LT model to the influence maximization problem in the IC model.

To find a relation between the LT model and IC model, we construct a new instance G' in the IC model, but the graph G' is the same as G both in structures and weights of edges. The following lemma is the technical lemma in our proof, which reveals the relation between the two models (Fig. 2).

Lemma 4. *$OPT_A^{LT}(G,k) \leq OPT_A^{IC}(G',2k)$*

Proof. The proof outline is to construct an algorithm for G' based on the optimal adaptive algorithm for G. There is an observation that after choosing the same

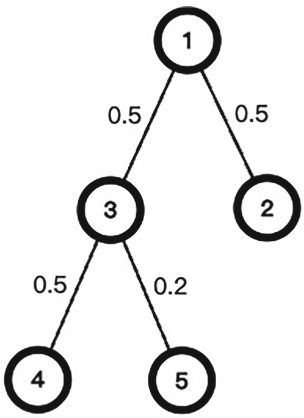

Fig. 2. This figure gives an example. In the first round, we choose vertex indexed 4, and the diffusion process stops at it self. In the second, round we choose vertex 5, while the process also stops at itself. According to our reduction, we have the probability $p_{5,3}(1/pt_3 - 1) = 0.2$ to add vertex 3 to our seed set in this case.

first seed vertex both in G and G', the diffusion process shares the same distribution on the in-arborescence. However, in the following process, the appearing probability of the edge would increase in the LT model, and we need a larger seed set in the IC model to compensate for the boosted probability.

To formalize the intuitions above, we design an k-round adaptive algorithm for G'. Let π be the optimal adaptive algorithm for G, and π' be the algorithm constructed for G'. First, we maintain a current partial realization ψ, which is an empty set at first. In the first round, we simulate the algorithm π in G' to get the first seed of π'. In π', we also add $\pi(\psi)$ ($\psi = \emptyset$ at first) to our seed set, and add $(\pi(\psi), R(\pi(\psi)))$ to ψ. Suppose that the diffusion process of $\pi(\psi)$ end at vertex u, we mark the parent of u (if exists) as a *critical point*. Also, we maintain a value for each vertex v called *remaining potential* defined as

$$pt_v := \sum_{u \in \text{Child}(v), u \text{ is not activated}} p_u,$$

where we use Child(v) to denote the set of children of v on the tree.

In the next $k-1$ rounds, we first choose $x = \pi(\psi)$ to be our new seed. However, the existing probabilities of some edges increased because of the previous rounds. Suppose the diffusion process of x stops at a vertex u, and let its parent be v. If v is a critical point, which means the existing probability of edges (u, v) has already increased, we flip a biased coin which appears heads with probability $p_{u,v}(1/pt_v - 1)$. If the coin appears heads, we choose v to be our next seed, remove v from critical vertex sets, and continue this round. On the contrary, we just go to the next round. Obviously, this process eventually ends at some vertex y. Before the end of this round, we update ψ with $\psi \cup (x, \{\text{the paths from } x \text{ to } y)\}$, and mark y's parent (if exists) as a new critical point.

First of all, it's easy to verify that, the ψ after i rounds shares the same distribution in the LT model after choosing the first i seeds. Thus, $\mathbf{E}_{\mathrm{LT}}(\pi) = \mathbf{E}_{\mathrm{IC}}(\pi')$. Also, there is an observation that there are at most k vertices marked as critical points. Thus, we have that

$$|\pi'| \leq k + k = 2k,$$

and

$$\mathrm{OPT}_A^{\mathrm{LT}}(G, k) = \mathbf{E}_{\mathrm{LT}}(\pi) = \mathbf{E}_{\mathrm{IC}}(\pi') \leq \mathrm{OPT}_A^{\mathrm{IC}}(G', 2k),$$

as desired.

Lemma 4 builds a connection between the two models. Further analysis is needed to give an upper bound of AG_{LT}.

Lemma 5. *$OPT_N^{LT}(G, k) \geq OPT_N^{IC}(G', k)$*

Proof. This can be proved by an easy reduction. We want to prove that for every fixed seed set S, it holds that,

$$\mathbf{E}_{L\sim\mathrm{LT}}(R(L, S)) \geq \mathbf{E}_{L\sim\mathrm{IC}}(R(L, S)).$$

First, by the linearity of the expectation, it holds that,

$$\mathbf{E}_{L\sim\mathrm{LT}}(R(L, S)) = \sum_{v\in V} \Pr_{L\sim\mathrm{LT}}[v \text{ is activated}].$$

Similarly, we have that,

$$\mathbf{E}_{L\sim\mathrm{IC}}(R(L, S)) = \sum_{v\in V} \Pr_{L\sim\mathrm{IC}}[v \text{ is activated}].$$

Then, we will prove by induction that

$$\Pr_{L\sim\mathrm{LT}}[v \text{ is activated}] \geq \Pr_{L\sim\mathrm{IC}}[v \text{ is activated}]$$

with a decreasing order of v's depth.

For a vertex v with the largest depth, if it is in the seed set, it holds that

$$\Pr_{L\sim\mathrm{LT}}[v \text{ is activated}] = \Pr_{L\sim\mathrm{IC}}[v \text{ is activated}] = 1.$$

Otherwise, it holds that

$$\Pr_{L\sim\mathrm{LT}}[v \text{ is activated}] = \Pr_{L\sim\mathrm{IC}}[v \text{ is activated}] = 0.$$

For a vertex w of another depth, assume that every child of w satisfies the induction hypothesis. We have that,

$$\begin{aligned}\Pr_{L\sim\mathrm{LT}}[w \text{ is activated}] &= \sum_{u\in\mathrm{Child}(v)} p_{u,w} \Pr_{L\sim\mathrm{LT}}[u \text{ is activated}] \\ &\geq \sum_{u\in\mathrm{Child}(v)} p_{u,w} \Pr_{L\sim\mathrm{IC}}[u \text{ is activated}] \\ &\overset{\text{Union bound}}{\geq} 1 - \prod_{u\in\mathrm{Child}(v)} (1 - p_{u,w} \Pr_{L\sim\mathrm{IC}}[u \text{ is activated}]) \\ &= \Pr_{L\sim\mathrm{IC}}[w \text{ is activated}],\end{aligned}$$

as desired.

The last step is to bound $\mathrm{OPT}_A^{\mathrm{IC}}(G', 2k)$ by $\mathrm{OPT}_A^{\mathrm{IC}}(G', k)$. This comes from the following submodularity lemma:

Lemma 6 (Adaptive Submodularity for the IC model, [GK11]**).** *Let G be an arbitrary influence graph. For any partial realizations ψ, ψ' of G such that $\psi \subseteq \psi'$, and any node $u \notin R(\psi')$, we have that $\Delta(u \mid \psi') \leq \Delta(u \mid \psi)$, where $\Delta(u \mid \psi)$ represents the expected increasing influence to choose u under ψ.*

This lemma gives a good property of the IC model, leading to the following submodularity lemma of the optimal adaptive algorithm:

Lemma 7. *$OPT_A^{IC}(G', 2k) \leq 2OPT_A^{IC}(G', k)$*

Proof. First, we divide the optimal adaptive algorithms π' for G' with a fixed seed set size $2k$. We want to argue that the expected influence of each part is less than $\mathrm{OPT}_A^{\mathrm{IC}}(G', k)$.

For the first part, it is an adaptive algorithm with seed set size equaling k. Thus, the total influence should be not more than $\mathrm{OPT}_A^{\mathrm{IC}}(G', k)$.

After the selection of the first k seeds, there exists a non-empty partial realization ψ. We want to prove that if we select k more seeds, the expected extra influence is no more than $\mathrm{OPT}_A^{\mathrm{IC}}(G', k)$. This is a natural corollary of adaptive submodularity.

Thus, we have $\mathrm{OPT}_A^{\mathrm{IC}}(G', 2k) \leq 2\mathrm{OPT}_A^{\mathrm{IC}}(G', k)$ as desired.

Lemma 8 ([DPV23]). $AG_{\mathrm{IC}} \leq \frac{2e^2}{e^2-1}$.

This bound is given by [DPV23]. And can be used to give a bound of AG_{LT}:

Proof (Proof of Theorem 1) Putting together Lemma 4, 5, 7, 8, we have

$$\begin{aligned}
AG_{\mathrm{LT}} &= \sup_{G,k} \frac{\mathrm{OPT}_A^{\mathrm{LT}}(G,k)}{\mathrm{OPT}_N^{\mathrm{LT}}(G,k)} \\
&\leq \sup_{G,k} \frac{\mathrm{OPT}_A^{\mathrm{LT}}(G,k)}{\mathrm{OPT}_N^{\mathrm{IC}}(G',k)} \\
&\leq \sup_{G,k} \frac{\mathrm{OPT}_A^{\mathrm{IC}}(G',2k)}{\mathrm{OPT}_N^{\mathrm{IC}}(G',k)} \\
&\leq \sup_{G,k} \frac{2\mathrm{OPT}_A^{\mathrm{IC}}(G',k)}{\mathrm{OPT}_N^{\mathrm{IC}}(G',k)} \\
&\leq 2AG_{\mathrm{IC}} \\
&\leq \frac{4e^2}{e^2-1},
\end{aligned}$$

as desired.

4 Discussions and Open Questions

In [CP19] and [DPV23], they also give some constant upper bounds under the IC model for some other graphs such as one-directional bipartite graphs. Also, [DPV23] gives an upper bound for general graphs though it is not a constant bound. We have the following conjecture.

Conjecture 1 The adaptivity gap for general graphs under the LT model has a constant upper bound.

However, adaptivity algorithms gain more profits in the LT model than the IC model. Thus, we also have the following conjecture.

Conjecture 2 The adaptivity gap for general graphs under the IC model has a constant upper bound.

Also, we have a conjecture about the relation between the two models on the general graphs, and we believe that our approach can help to build a similar argument on general graphs.

Conjecture 3 There is a Lemma 4 like argument holds for general graphs.

Thus, as a corollary of the above conjecture, we are able to claim that the adaptivity gap on LT model is linearly upper bounded by the adaptivity gap on IC model.

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